

**Conference on Lasers and Electro-Optics  
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Session: II-2.6 / Plasmonics Antennas and Waveguides

# **Properties of Highly-Nonlinear Hybrid Silicon-Plasmonic Waveguides**

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## Presentation Outline

### □ Introduction

- Nonlinear Effects in Plasmonics
- Objective, Motivation & Challenge

### □ Modeling Propagation in Nonlinear Waveguides

- Nonlinear Schrodinger Equation (NLSE)
- Derivation of Figures-of-Merit (FoM) in CW

### □ Hybrid Silicon-Plasmonic Waveguides

- Review of Hybrid Silicon-Plasmonic (HSP) Waveguides
- Optimization of HSP Waveguides for Kerr-type Applications
- Comparison to Dominant SOI-based Platforms
- Nonlinear Directional Coupler – Design & Applications

### □ High-Power Illumination in Nanophotonic Waveguides

- An Unexplored Operation Regime – Accessibility and Modeling
- Prospects & Considerations

### □ Conclusion & Future Perspectives

Nonlinear Effects and Silicon-Plasmonics

# Introduction

## Introduction: Nonlinear Effects

Effects **originating** from **material  $\chi^{(3)}$**  the (3<sup>rd</sup> order nonlinear susceptibility)

❖ **Magnitude** is **proportional to optical intensity**  $I=|\mathbf{E}|^2$ .

❖ **Potential** for **all-optical functionality**, i.e. “*light controlling light*”

❖ **Focus** only in **intra-band interactions**, i.e.  $\Delta\omega \ll \omega_0$

□ **Ultrafast/Instantaneous** response:

- **Kerr Effect** → refractive index perturbation
- **Two-Photon Absorption (TPA)** → attenuation + carrier generation

□ **Delayed/Resonant** response:

- **Free-Carrier Effects (FCE)** → From TPA in semiconductors
- ❖ **Raman Scattering** → Interaction with optical phonons

### Kerr-effect in NL-waveguides:

- ✓ Optical power (low)
- ✓ Material nonlinearity (high)
- ✓ Attenuation (low)
- ✓ Footprint (small)
- ❖ Phase matching

### Free-Carrier Effects

- Depend on FC-density
- Induce additional...
  - ❖ attenuation → limits power
  - ❖ dispersion → “masks” Kerr
- More critical than TPA

## Introduction: Plasmonics

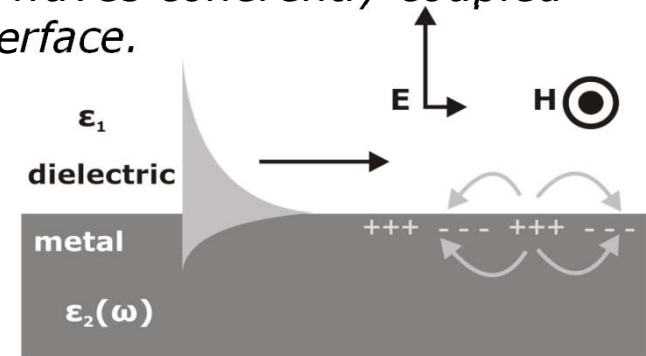
**Surface Plasmon Polaritons (SPPs):** EM surface waves coherently coupled to free electron oscillations on a metal/dielectric interface.

**Metal at NIR → Drude model:**  $\text{Re}\{\epsilon_2\} < 0$

- SPP waves propagate along the interface.
- Fields decay exponentially away from it.

**Trade-off → losses vs. lateral confinement**

- ✗ Suffer ohmic propagation losses (metal).
- ✓ Confinement surpasses Diffraction Limit.



*SPP at single metal/dielectric interface:  
An elementary plasmonic waveguide*

### **Plasmonics for Optical Communications**

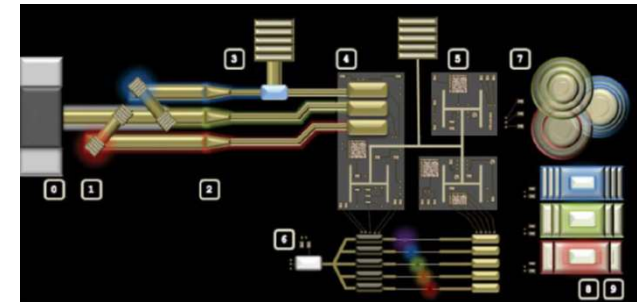
**Integrated photonic components** with...

- ✓ **Lateral dimensions** → far-below diffraction limit ( $\lambda/2$ )
  - ✓ **Control & Information** signals collocated @ metal/dielectric interface
- ...leading to **Nanoscale Opto-Electronic Devices**.

## Scope: Guided-Wave Nonlinear Plasmonics

**Objective:** Design of novel integrated Components for optical communications with:

- ✓ **all-optical functionality**
- ✓ **minimal footprint & interaction lengths**
- ✓ **reduced power threshold** for Kerr
- ✓ **limited FCE impairments**



*Future integrated plasmonic circuit  
(Dionne et al, 2010)*

**Motivation:** High confinement → High waveguide nonlinearity

... leading to: **smaller interaction lengths & reduced power**

❖ **Plasmonic waveguides can truly excel in this aspect!**

**Challenge:** Counterbalance the **inherent ohmic losses** in plasmonic devices

**Opportunities & Prospects:** Towards efficient **nonlinear plasmonics**

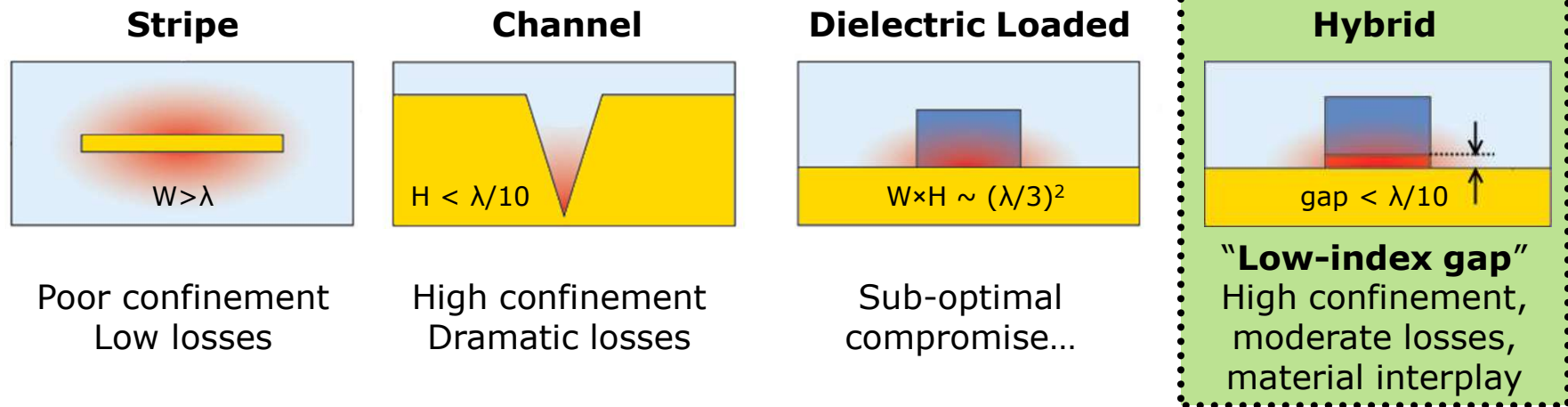
- ❑ **Synergy** with dominant **silicon-photonics**
- ❑ **Exploitation** of novel materials such as **highly-NL polymers**

From Photonics to Silicon-Plasmonics

# Hybrid Silicon-Plasmonic (HSP) Waveguides

# Plasmonic Waveguides for Nonlinear Applications

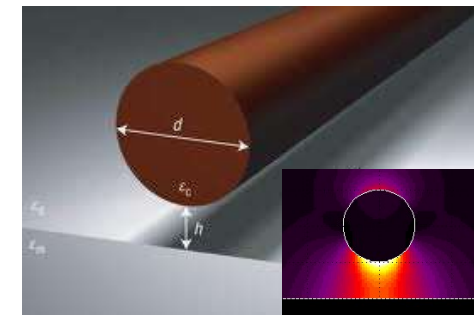
## Overview of SPP Waveguides (Berini & De Leon, Nat. Phot., 2012)



## Advantages & Prospects:

- ✓ Gap material → **nonlinear** (or electro-optic)
- ✓ **Small field penetration in Silicon** → low TPA & FCE
- **Carrier-sweeping circuit** (Si-slab & electrodes)
- Planar → **Easy fabrication** (lithography)
- **Efficient coupling** to SOI-waveguide
- **Thermal exhaustion** (through Si- and metal-slab)

Oulton *et al.*, 2008,  
Nature Photonics

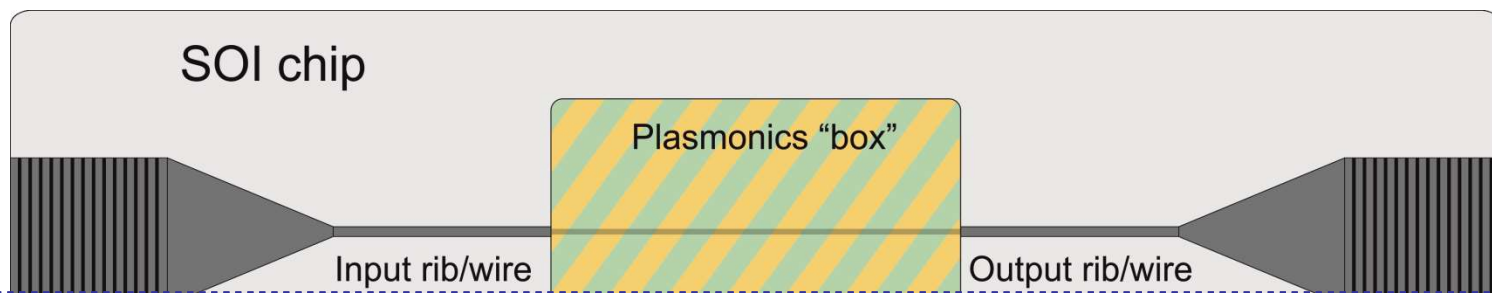




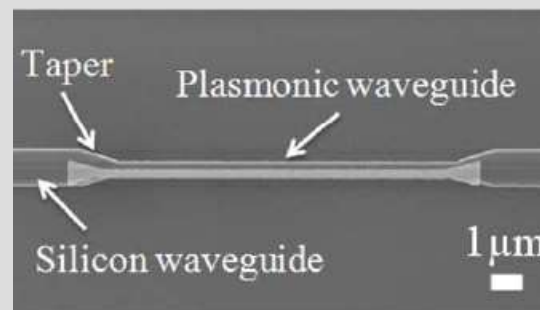
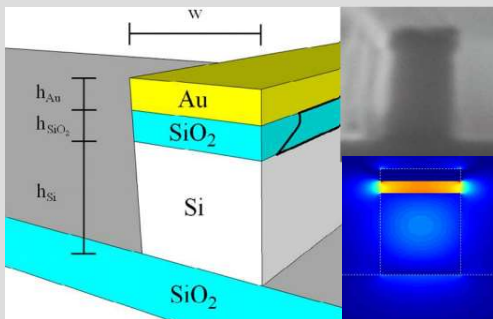
## Merging Silicon-Photonics & Plasmonics

### Integration on SOI motherboard:

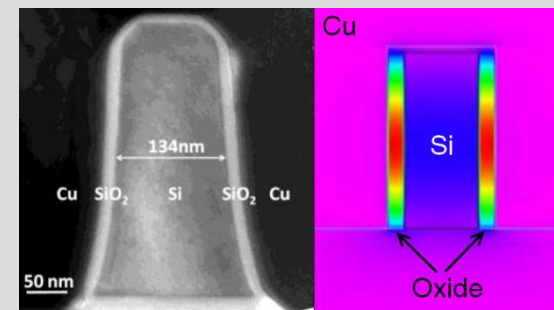
- **Host** for both HSP- and Si-waveguides.
- Provides **“seamless” interface** w/ silicon photonics at minimal losses.
- **Metals** → Au, Ag or Cu (CMOS-friendly).



Wu *et al.*, 2010, Optics Express. **CGS: “Conductor-Gap-Silicon”**



Zhu *et al.*, 2011, Optics Express  
**“Plasmonic nano-slot”**



Modeling Propagation in Si-comprising NL waveguides

# **Nonlinear Schrödinger Equation + Figures-of-Merit**

# The Nonlinear Schrodinger Equation (NLSE)

Models slowly-varying envelope propagation along the z-axis:  $A(z, t)$

$$\frac{\partial A}{\partial z} = \left[ -\frac{a}{2} A + \left( \sum_{n=1}^{\infty} \beta_n \frac{i^{n+1}}{n!} \frac{\partial^n}{\partial t^n} \right) A + i\gamma_c |A|^2 A + i\delta_{fc} A \right] + \left[ \frac{d}{dt} \langle N \rangle = \langle G \rangle - \frac{1}{\tau_{fc,eff}} \langle N \rangle \right]$$

FC-rate eq.

□ **Attenuation**: Ohmic loss, confinement, surface-roughness, ...

□ **Dispersion**: From material & waveguide engineering → Vanishes in CW

□ **Nonlinearity**: Instantaneous, complex-valued → SPM & TPA

□ **Free-Carrier Effects**: TPA@Silicon → Dispersion & Absorption + **Dynamics**

$$\chi_{xxxx}^{(3)} = \frac{4}{3} c_0 \epsilon_0 \epsilon_r \left( n_2 + i \frac{\beta_{TPA}}{2k_0} \right)$$

$$\mathcal{N} \triangleq \text{Re} \left\{ \iint (\mathbf{e} \times \mathbf{h}^*) \cdot \hat{\mathbf{z}} dx dy \right\}$$

$$\gamma_c = \frac{3\omega\epsilon_0}{4\mathcal{N}^2} \sum_{\mu,\alpha,\beta,\gamma}^{x,y,z} \iint \chi_{\mu\alpha\beta\gamma}^{(3)} e_{\mu}^* e_{\alpha} e_{\beta}^* e_{\gamma} dx dy$$

$$\delta_{fc} = \left( k_0 n_{Si} \Gamma / n_{eff} \right) \langle \Delta u_{fc} \rangle$$

$$\Delta u_{fc} = [\Delta n_{fc} + (i / 2k_0) \Delta a_{fc}] \approx \sigma_u N$$

$$\langle \Delta u_{fc} \rangle \triangleq \iint \Delta u_{fc} |\mathbf{e}|^2 dx dy / \iint |\mathbf{e}|^2 dx dy$$

$$G = \frac{1}{2\hbar\omega_0} \left( \frac{\partial P_{TPA}}{\partial z} \right) = \frac{1}{2\hbar\omega_0} \left( \beta_{TPA} \tilde{P}_z^2 |A|^4 \right)$$

## Simplifying the NLSE for CW

**Time-derivatives** → (1) Dispersion terms and (2) FC-rate Equation

$$\frac{\partial A}{\partial z} = \left[ i \left( \gamma |A|^2 - f_D |A|^4 \right) - \frac{1}{2} \left( a + 2r\gamma |A|^2 + 2f_A |A|^4 \right) \right] A$$

Kerr
FCD
Loss
TPA
FCA

{

**Loss**  $a \triangleq 1 / L_{\text{prop}}$

**Kerr**  $\gamma \triangleq \text{Re}\{\gamma_c\}$

**TPA**  $r \triangleq \text{Im}\{\gamma_c\} / \text{Re}\{\gamma_c\}$

**FCD**  $\left[ f_D \right] = \frac{\Gamma n_{\text{Si}}}{n_{\text{eff}}} \times \Xi \tau_{\text{fc}} \times k_0 \times \left[ \begin{matrix} \text{Re}\{-\sigma_u\} \\ \text{Im}\{+\sigma_u\} \end{matrix} \right]$

**FCA**

**NLSE parameters calc.:**  
→ FEM Eigenmode Solver

}

### Importance of Free-Carrier Effects:

- FCA scales with  $|A|^4$  → Faster than TPA
- FCD has an opposite sign from Kerr

Longitudinal Enhancement

$$\Gamma \triangleq \frac{c_0 \epsilon_0 n_{\text{eff}}}{\mathcal{N}} \iint |\mathbf{e}|^2 dx dy$$

$\Gamma \geq 1$  → mode deviation from TEM

Free-Carrier complex  
Index-Perturbation

$$\sigma_u \triangleq \left[ -\sigma_n + (i / 2k_0) \sigma_a \right]$$

Soref & Bennett → Drude fitting

Carriers/Field overlap

$$\Xi \triangleq \left\langle \beta_{\text{TPA}} \tilde{P}_z^2 \right\rangle \text{ m}^{-3} \text{ W}^{-1}$$

Spatial matching of FC & field

## Figure-of-Merit (FoM) Derivation

**Conventional nonlinear waveguides** → Kerr and Linear-losses only

- Nonlinear phase-shift:  $\Phi_{\text{Kerr}} = \gamma P_{\text{in}} L_{\text{eff}}$  and  $L_{\text{eff}} \equiv 0.6321 L_{\text{pr}} @ L = L_{\text{pr}}$

❖ Basic **Figure-of-Merit FoM**:  $\mathcal{F} \triangleq \gamma / a = \gamma L_{\text{prop}}$  (in 1/Watt)

- ✓ Kerr-related effects (FWM, SPM, XPM etc) need a power level  $\propto 1 / \mathcal{F}$

**Silicon-comprising waveguides** → FCD & TPA+FCD affect the phase & loss

- CW-NLSE inspection:  $\gamma' \rightarrow \gamma - f_D |A|^2$ ,  $a' \rightarrow a + 2r\gamma |A|^2 + 2f_A |A|^4$
- A power-dependent FoM  $\mathcal{F}' \triangleq \gamma' / a'$  ← but, useful when power is given...

**Threshold-power**: FCD-equal-to-Kerr →  $\gamma' \equiv 0 \Rightarrow P_{\text{th,FCD}} \triangleq \gamma / f_D = \zeta_{\text{fc}} \times \varpi$

where  $\varpi \triangleq 2\hbar c_0 / (\tau_{\text{fc}} n_{\text{Si}} \sigma_n)$  in (W/m<sup>2</sup>) → ~constant (for a given w/g design)

❖ **Kerr-vs-FCE FoM**:  $\zeta_{\text{fc}} \triangleq \gamma n_{\text{eff}} / (\Xi \Gamma)$  (in m<sup>2</sup>) → major contribution  $\gamma / \Xi$

- ✓ **Overall**:  $1 / \mathcal{F} \leq P_{\text{th,FCD}} \rightarrow \boxed{\mathcal{F} \times \zeta_{\text{fc}} \times \varpi \geq 1}$  **for FCE-free operation (CW)**

## \* Multimode Waveguides: Coupled NLSE System

Waveguide supports  $K$  modes → A **system of  $K$  differential NLSE**

**Coupling** via (1) Kerr/TPA and (2) free-carrier effects

$$\frac{\partial A_k}{\partial z} = -\frac{a_k}{2} A_k + \left( \sum_{n=1}^{\infty} \beta_n^{(k)} \frac{i^{n+1}}{n!} \frac{\partial^n}{\partial t^n} \right) A_k + i \sum_{l,m,n}^K \gamma_{klmn} A_l A_m^* A_n e^{i\Delta\beta_{klmn}z} + i\delta_{kk} A_k$$

### Nonlinear parameters (Eigenmode)

$$\gamma_{klmn} = \frac{3\omega\epsilon_0}{4} \sum_{\mu,\alpha,\beta,\gamma}^{x,y,z} \iint \chi_{\mu\alpha\beta\gamma}^{(3)} \frac{e_{\mu}^{*(k)} e_{\alpha}^{(l)} e_{\beta}^{*(m)} e_{\gamma}^{(n)}}{\sqrt{\mathcal{N}_k \mathcal{N}_l \mathcal{N}_m \mathcal{N}_n}} dx dy$$

### Phase-matching for Kerr/TPA

$$\Delta\beta_{klmn} = -\beta_0^{(k)} + \beta_0^{(l)} - \beta_0^{(m)} + \beta_0^{(n)}$$

**FCE perturbation → depends on  $\langle N \rangle_k$  FC-density**

$$\delta_{kk}(z, t) = \left( k_0 n_{\text{Si}} \Gamma_{(k)} / n_{\text{eff}}^{(k)} \right) \sigma_u \langle N \rangle_k$$

**FC-generation Rate  $\langle G \rangle_k$  for each  $k$ -mode**

$$\langle G \rangle_k = \frac{1}{2\hbar\omega_0} \sum_{m,n=1}^K \Xi_{mn}^{(k)} |A_m|^2 |A_n|^2$$

**Density  $\langle N \rangle_k$  for each  $k$ -mode**

$$\frac{d}{dt} \langle N \rangle_k = \langle G \rangle_k - \frac{1}{\tau_{\text{fc,eff}}^{(k)}} \langle N \rangle_k$$

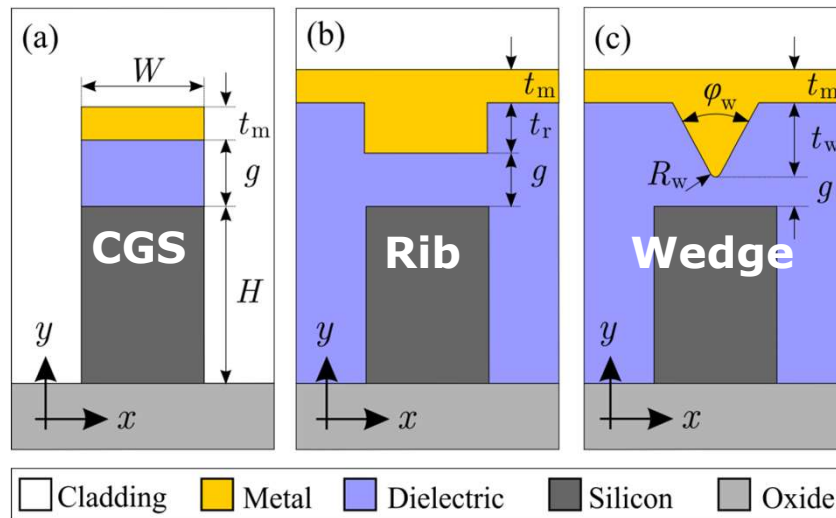
**Generalized carrier-field spatial overlap**

$$\Xi_{mn}^{(k)} \triangleq \left\langle \beta_{\text{TPA}} \tilde{P}_z^{(m)} \tilde{P}_z^{(n)} \right\rangle_k$$

Kerr-type Nonlinear Applications & low FCE impairments

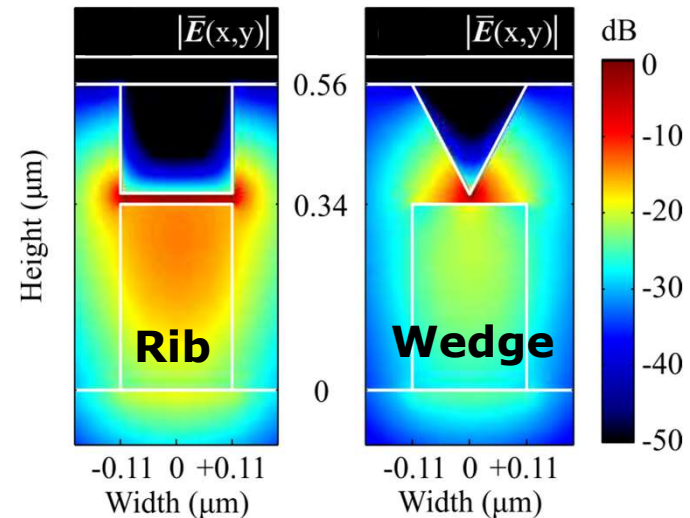
# Optimizing the HSP waveguide

# Optimization of HSP waveguides (1/3)



## Materials

- Metal → **Silver** with  $n_0 = 0.145 - j11.4$
- Gap → **DDMEBT** nonlinear polymer  
 $n_2 = 1.7 \times 10^{-17} \text{ m}^2/\text{W}$ ,  $n_0 = 1.8$
- Silicon →  $\beta_{\text{TPA}} = 5 \times 10^{-12} \text{ m/W}$ ,  
 $n_2 = 2.5 \times 10^{-18} \text{ m}^2/\text{W}$ ,  $n_0 = 3.45$ ,  
**Carrier-Lifetime:**  $\tau_{\text{fc}} = 1 \text{ nsec}$   
 $\varpi \sim 10^{10} \text{ W/m}^2$  (as low as 10 ps)



## Targeted performance @ $\lambda = 1.55 \mu\text{m}$

- ✓  $\gamma_{\text{NL}} > 10\,000 \text{ m}^{-1}\text{W}^{-1}$
- ✓  $A_{\text{eff}} < 0.01 \mu\text{m}^2$
- ✓  $L_{\text{prop}} > 30 \mu\text{m}$
- ✓  $r_{\text{TPA}} < 0.1\%$  + lowest possible “Ξ”

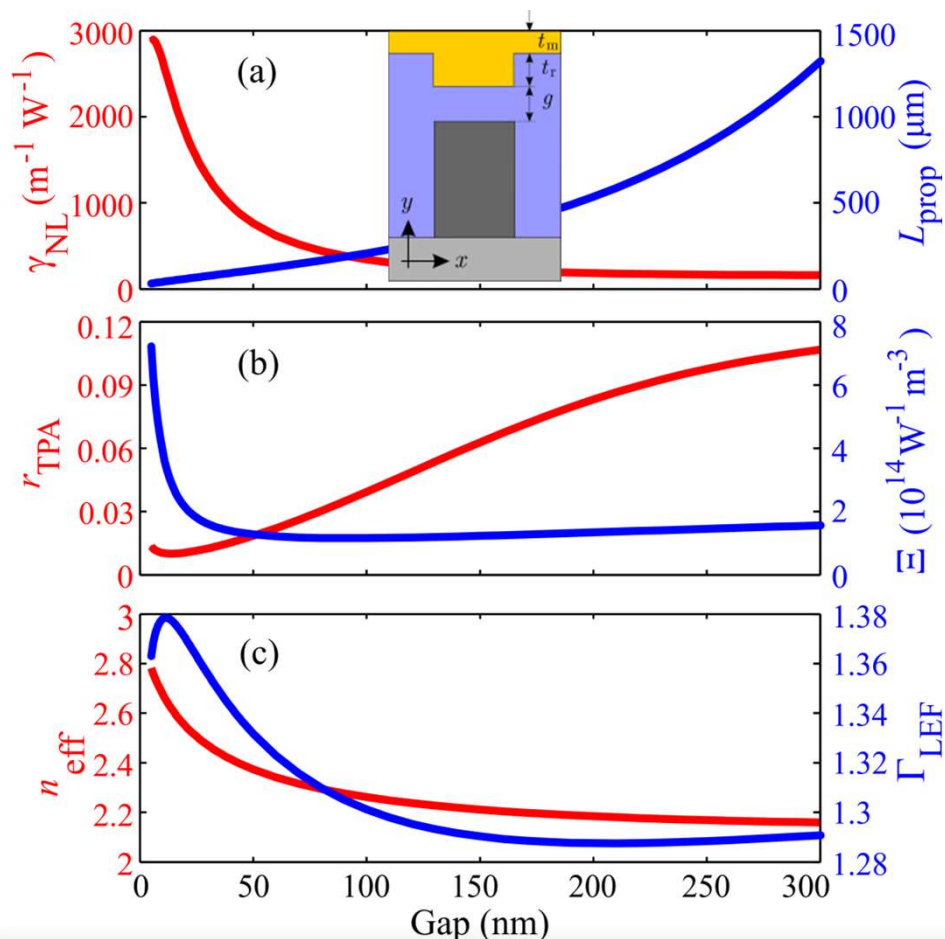
## Critical parameter: gap-size

- ❖ tech/fab limitation of  $g \geq 20 \text{ nm}$



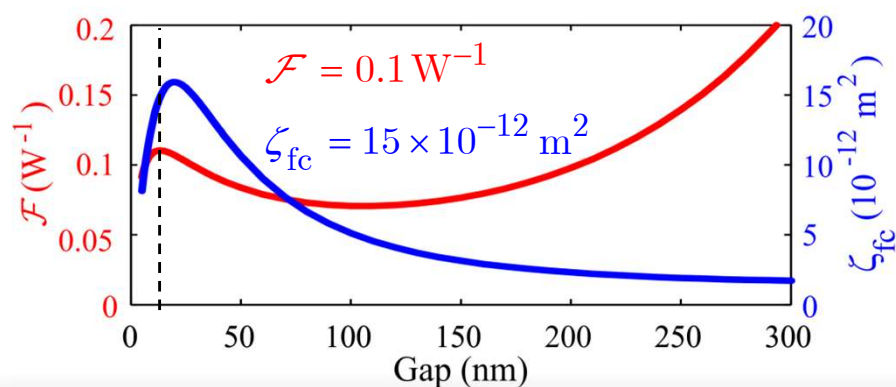
## \* Optimization of HSP waveguides (2/3) – Gap Effect

**HSP w/g mode parameters**

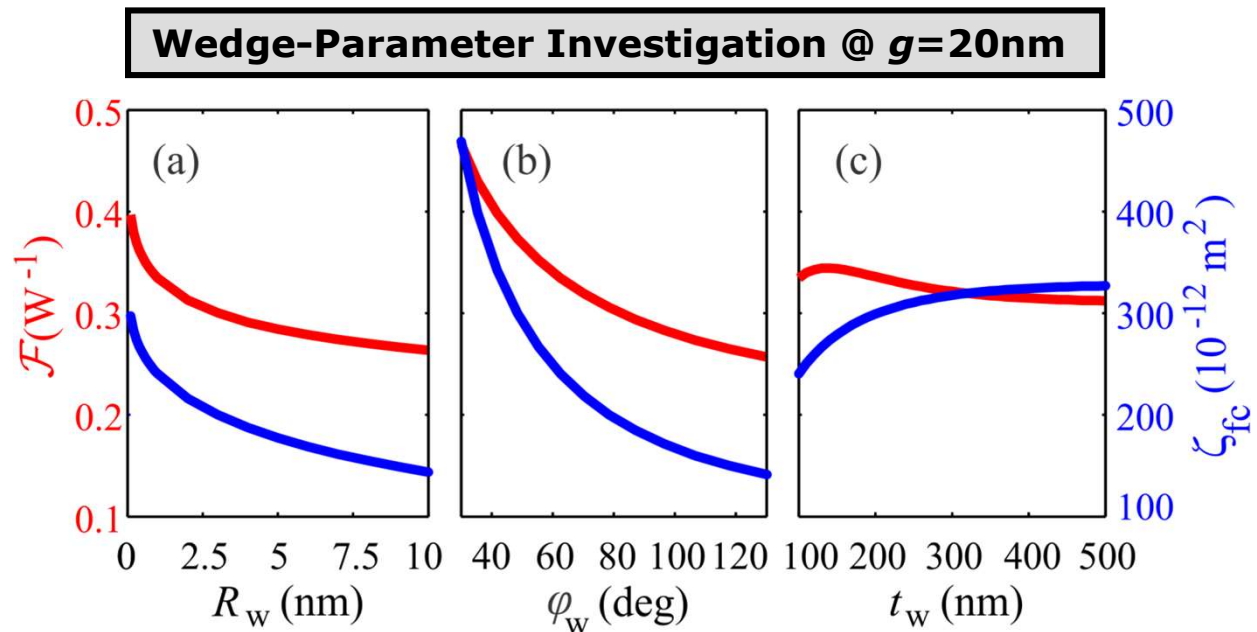
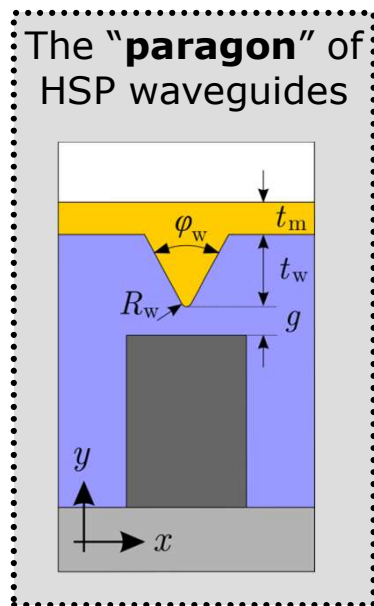


- ❑ **Trade-off:** confinement vs. loss
- ❑ **Large gaps:** modes degenerate into conventional Si-modes.
- ❑ **Small gaps:**
  - $r_{TPA}$  is considerably suppressed
  - $\Xi$  increases for  $g < 20 nm$ .
  - High  $\gamma_{NL} \sim 2000 m^{-1} W^{-1}$

**Figures-of-Merit**



## Optimization of HSP waveguides (3/3) : Inverted-Wedge



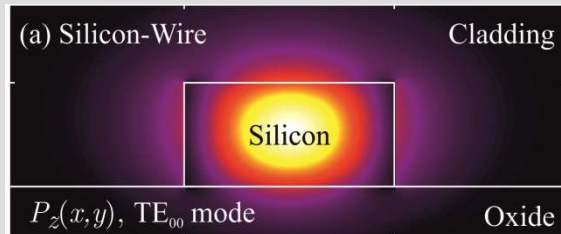
Significant **improvement in both FoM**  $\rightarrow \mathcal{F} \approx 0.4/W$  and  $\zeta_{fc} \approx 300 \times 10^{-12} \text{ m}^2$

- ✓ **Acute angles** provide better performance.
- Weak dependence on **tip-radius** and **wedge height**.
- Marginal dependence on **Si-wire dimensions** and **lateral misalignments**
- ❖ A wedge in uniform DDMEBT provides an order of magnitude smaller FoM.

## Comparison with prominent Si-comprising waveguides

### Conventional Si-wire

400x340 nm<sup>2</sup>



$$\mathcal{F} > 1 \text{ W}^{-1}$$

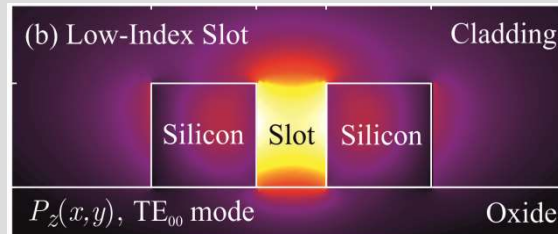
Length - scale ~ cm

$$\mathcal{F} \times \zeta_{fc} \times \varpi \approx 0.05$$

- ✓ Highest  $L_{prop}$
- ✗ Lowest  $\gamma_{NL}$
- ✗ **Lowest** FCE-threshold

### NL-Slot, width=140nm

(Koos, Nat. Photonics 2009)



$$\mathcal{F} \approx 0.5 \text{ W}^{-1}$$

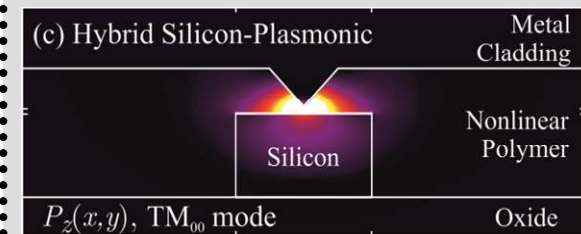
Length - scale ~ mm

$$\mathcal{F} \times \zeta_{fc} \times \varpi \approx 0.5$$

- Moderate  $L_{prop}$
- High  $\gamma_{NL}$
- High FCE-threshold

### Wedge-HSP

(This work)



$$\mathcal{F} \approx 0.4 \text{ W}^{-1},$$

Length - scale ~ 50μm

$$\mathcal{F} \times \zeta_{fc} \times \varpi \approx 0.5$$

- ✗ Smallest  $L_{prop}$
- ✓ Highest  $\gamma_{NL}$
- ✓ Highest FCE-threshold

**HSP vs. Slot: comparable overall-performance @ 1/20 length**

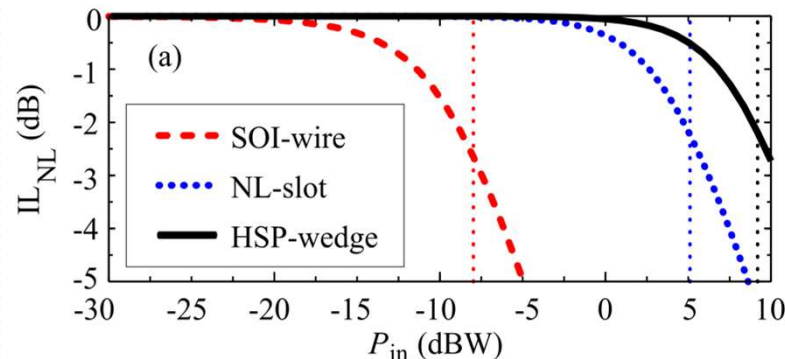
## \* Quantifying FCE power-thresholds (CW)

### FCE threshold-power

- Set  $L=L_{\text{prop}} \rightarrow$  different for each w/g
- Integrate CW-NLSE for increasing  $P_{\text{in}}$
- Quantify envelope phase and amplitude impairments  $\rightarrow A=|A|e^{i\Delta\Phi}$

#### NL-Threshold: FCA (and TPA)

$$IL_{\text{NL}} = |A|^2 \exp(L / L_{\text{prop}}) / P_{\text{in}}$$



$$P_{\text{th}}^{\text{FCA}} = \sqrt{a / f_A} \rightarrow IL_{\text{NL}} \approx -2.5 \text{ dB}$$

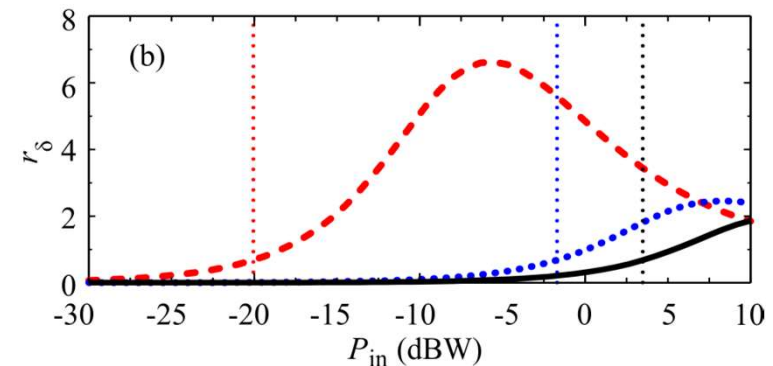
#### NL-Threshold: non-Kerr phase

$$\Delta\Phi_{\text{NL}} \triangleq -i \ln \left\{ \frac{A}{|A|} \right\} = \Phi_{\text{Kerr}} + \Phi_{\text{non-Kerr}}$$

$$r_{\delta} \triangleq \frac{-\Phi_{\text{non-Kerr}}}{\Phi_{\text{Kerr}}} = \frac{\gamma P_{\text{in}} L_{\text{eff}} - \Delta\Phi_{\text{NL}}}{\gamma P_{\text{in}} L_{\text{eff}}} > 0$$

$$\Phi_{\text{Kerr}} = \gamma P_{\text{in}} L_{\text{eff}} > 0 \quad \gamma \text{ and } L_{\text{eff}} \text{ from Linear case}$$

$$\Phi_{\text{non-Kerr}} < 0 \quad \text{From opposite FCD and } \Delta L_{\text{eff}} \text{ due TPA+FCA}$$



$$P_{\text{th}}^{\text{FCD}} = \gamma / f_D \rightarrow r_{\delta} \approx 0.7$$

Nonlinear HSP-waveguide based component

# Directional Coupler 2x2 Switch

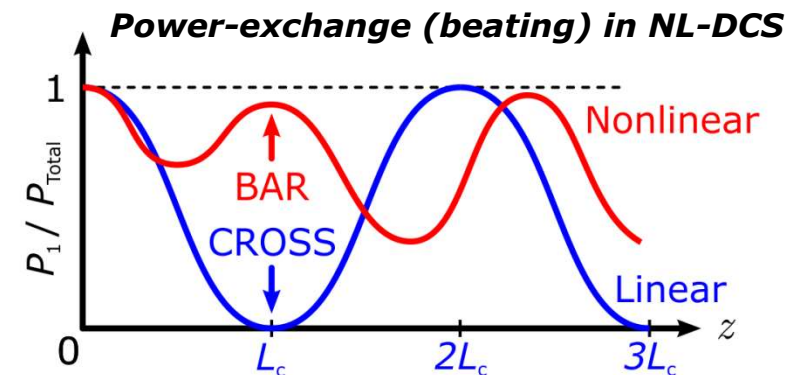
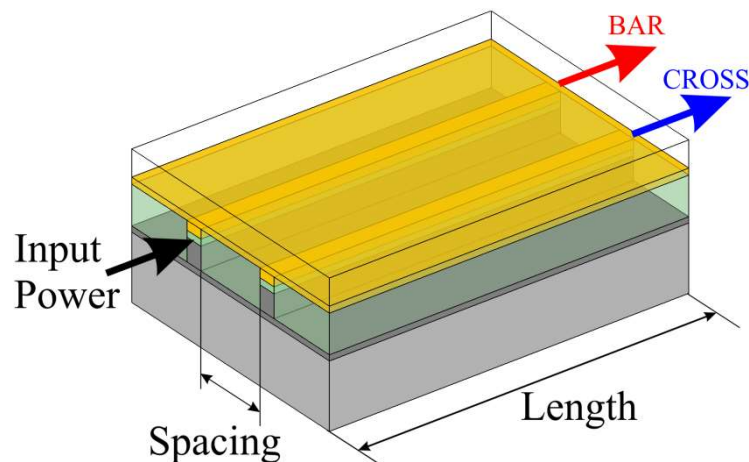
## The Nonlinear Directional Coupler Switch (NL-DCS)

**Directional Coupler** formed by a pair of HSP waveguides

- ❑ CROSS-state @ low input power → Linear regime
- ❑ BAR-state @ high input power → Nonlinear regime

### Operation Principle:

Self-focusing  
Induced coupler  
de-synchronization



Rough-estimate for **switching power**:

$$P_{sw} > \frac{\sqrt{3}\pi}{\mathcal{F}[1 - \exp(-L_c / L_{prop})]}$$

$$\Delta\beta_{Kerr}L_c > \sqrt{3}\pi$$

$$\Delta\beta_{Kerr}L_c \approx \Delta\Phi_{NL} = \gamma P_{in} L_{eff}$$

$$L_c = 0.5\lambda / \Delta n_{eff}^{(S-A)}$$



## \* NL-DCS: Symmetric & Anti-symmetric Supermodes

The NL-DCS can be analyzed in the context of

**multi-mode NLSE framework**

**HSP waveguide parameters:**

- Si-wire: 320x220nm<sup>2</sup>,
- DDMBER:  $g=20\text{nm}$
- Ag-wedge:  $t_w=100\text{nm}$ ,  $\phi_w=53.2^\circ$ ,  $R_w=1\text{nm}$
- Carrier Lifetime  $\tau_{fc}=0.1\text{nsec}$

Also supports  
TE modes!

**Performance metric** for switching:

$$\text{Output-port Crosstalk } XT = |A_L|^2 / |A_R|^2$$

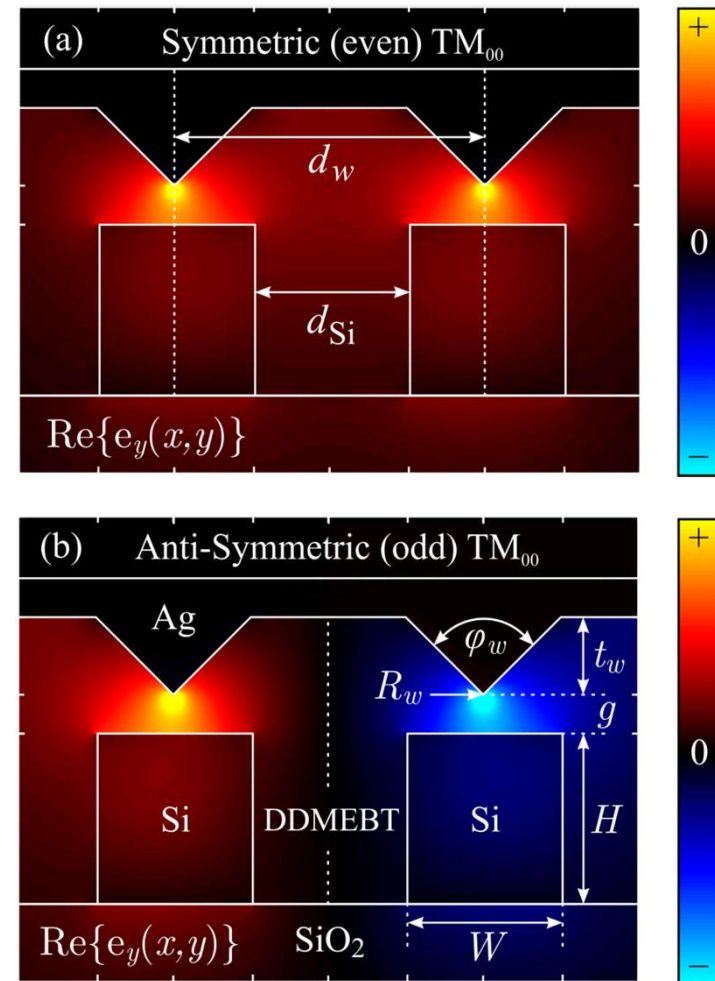
$$A_{L/R} = \left[ A_S \exp^{i\beta_0^{(S)}z} \pm A_A \exp^{i\beta_0^{(A)}z} \right] / \sqrt{2}$$

**Parametric Investigation:** w/g separation

$$d_{\text{Si}} = 280, 380, 480 \text{ nm}$$

$$L_c^{\text{TM}} = 14.6, 28.2, 54.0 \mu\text{m}$$

Linear  
Coupler



## NL-DCS: Switching Power

**Integrate coupled-NLSE system** for the three w/g separations.

□ XT vs.  $P_{in}$  at left input-port

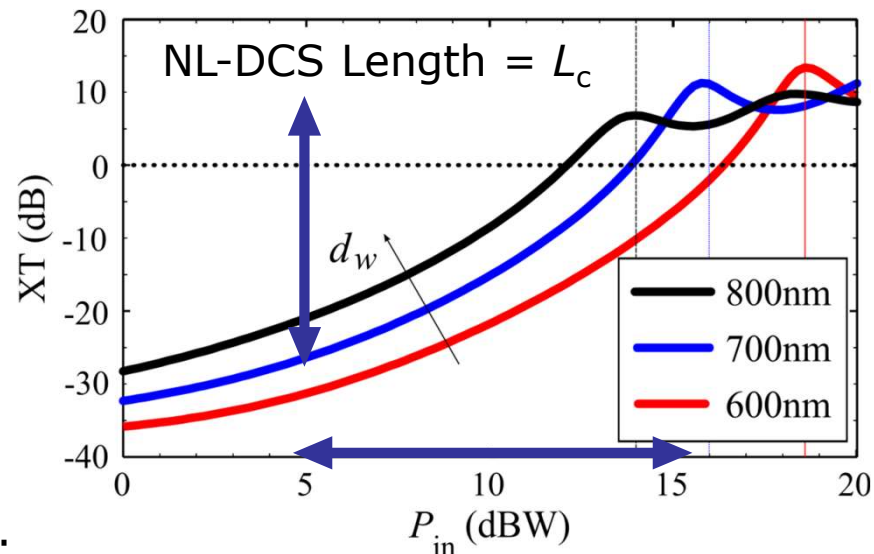
**Increasing w/g separation ( $d_w$ ):**

- ✓ reduces power @ 1<sup>st</sup> XT-peak ( $P_{sw}$ )
- ✗ increases component length ( $L_c$ ) → IL
- ✗ reduced XT values

**Comparison** with theoretical predictions:

$P_{in}$ @ max{XT}	
<i>empirical</i>	<i>simulation</i>
(dBW)	(dBW)
16	18.5
14	16
12.7	14

Power-penalty  
~2 dB  
due to FCE.



### Observation:

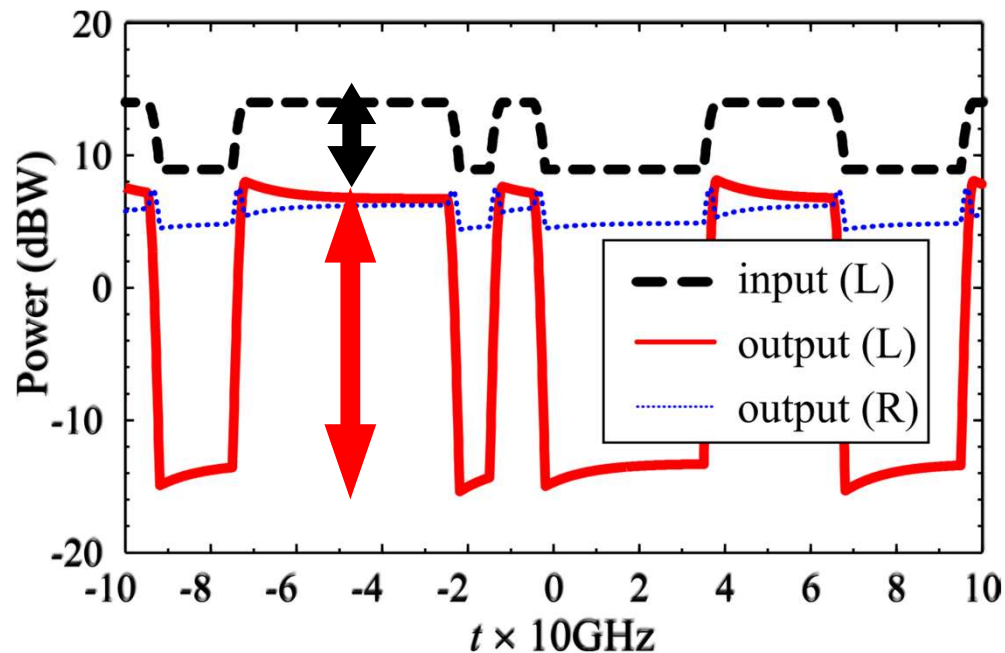
A  $P_{in}$ -change of 10 dB corresponds to an output-XT-change of >30 dB.

### Application:

Potential for improvement of the ER of modulated signals.



## NL-DCS: Boosting ER of Modulated Signals



### Modulated Input @ left-input

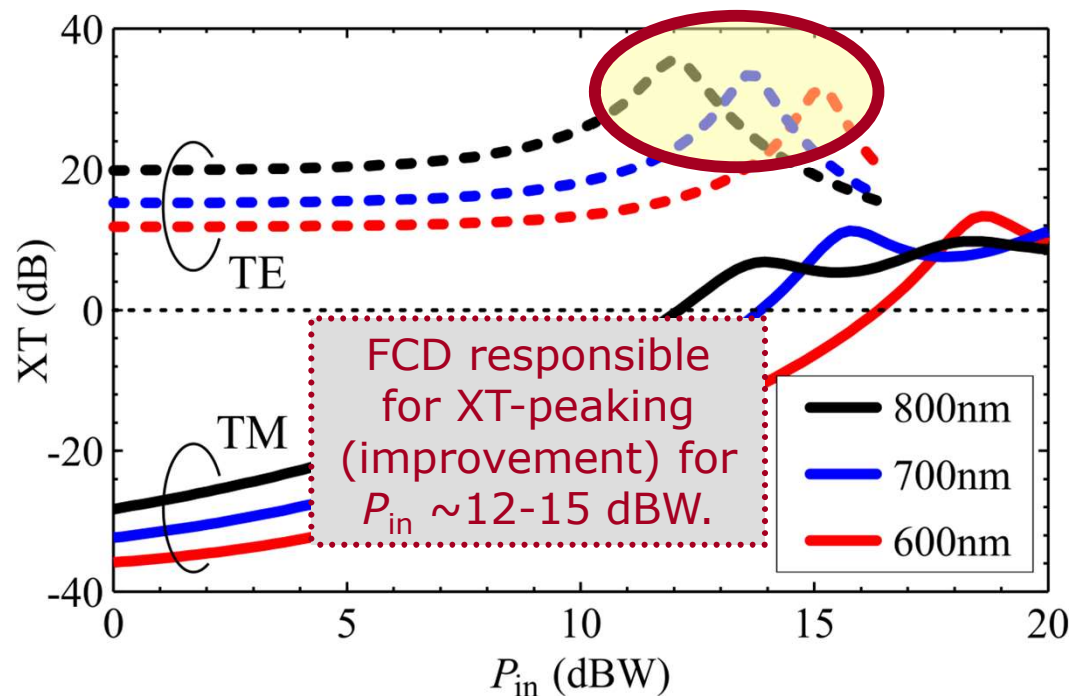
- 10Gbps NRZ
- TM-polarization
- 30ps rise/fall-time
- **ER=5dB**
- **$P_{\text{peak}}=14 \text{ dBW}$**

### HSP-wedge DCS:

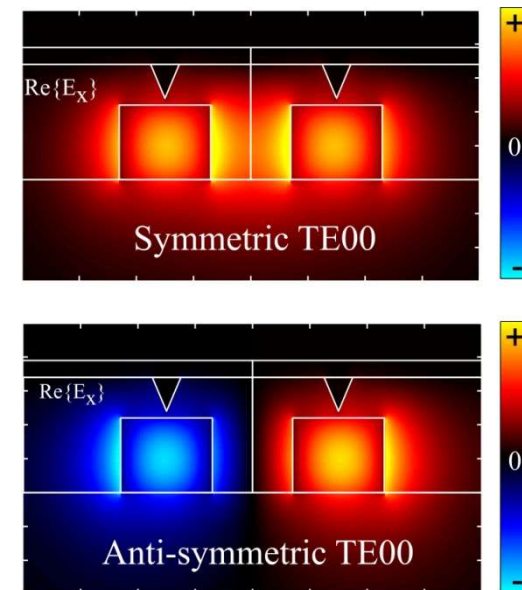
- $d_{\text{Si}}=380\text{nm}$  &  $L=28.2\mu\text{m}$

- ❖ **Numerical integration** (SSFM) of NLSE-pair for the S/A TM-supermodes.
- ❖ **Dispersive effects** → **negligible** for these length-scales
- ✓ **Output ER > 20dB** (15dB improvement) + penalty of IL  $\sim 4.5\text{dB}$ .

\* NL-DCS: TE modes



TE supermodes  
(photonic-like)



- $L_c(\text{TM}) / L_c(\text{TE}) \sim 2 \rightarrow$  DCS functions as **Polarization Splitter**
- NL-switching power is too high  $\rightarrow \text{XT} > 10\text{dB}$  for  $P_{in} < 20\text{dBW}$
- **Negligible polarization crosstalk** (TE/TM)  $< -40\text{dB}$

Extremely Nonlinear Waveguides

# High Power Illumination

## What happens?

**HSP waveguides: increased FCE-threshold** allows for **high peak-power**.

❖ **Inaccessible regime** for Si-core & NL-Slot waveguides

- Assuming: length-scales of  $L \sim L_{\text{prop}}$  and  $P_{\text{in}}$  below FCE-threshold

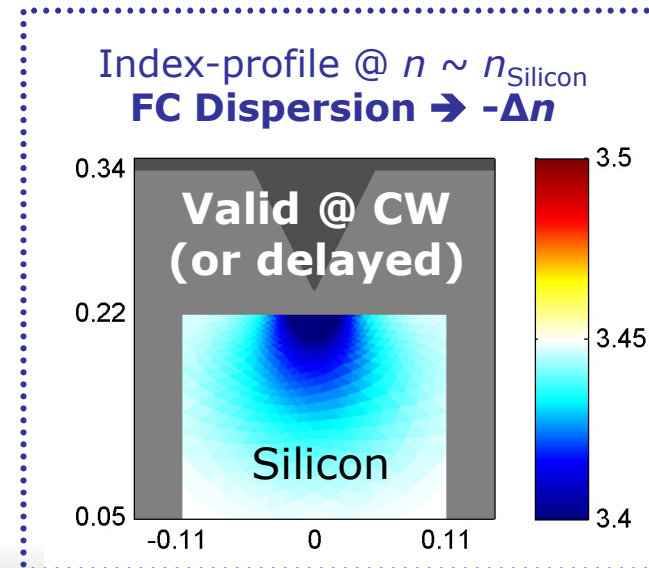
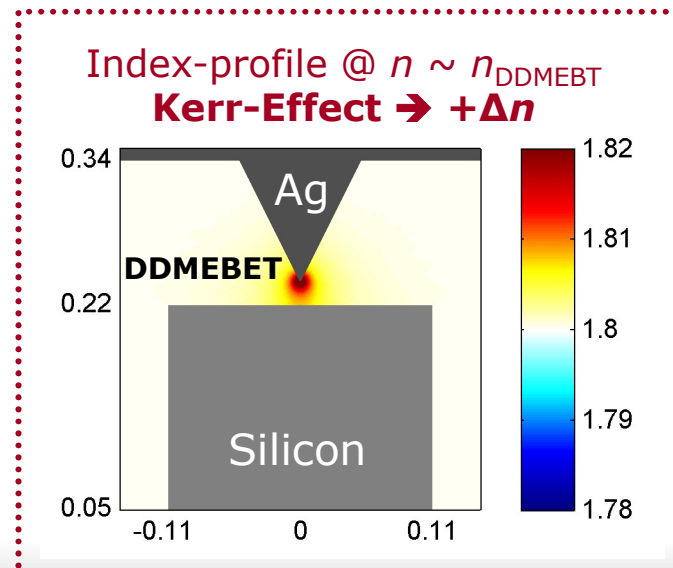
❖ **Instantaneous nature** of Kerr-type nonlinearity

**High-Power + High-nonlinearity + Instantaneous = ...**

- ❑ Considerable perturbation in the material refractive-index
- ❑ Eigenmode-profile is perturbed (for nano-sized w/g)
- ❑ Waveguide parameters (linear & nonlinear) are affected

**Become  
Power-dependent!**

$$\begin{matrix} n_{\text{eff}} & \& L_{\text{prop}} \\ \gamma_{\text{NL}} & \& r_{\text{TPA}} \\ \Xi_{\text{fc}} & \& \tau_{\text{fc,eff}} \end{matrix}$$



# Self-Consistent Eigenmode Solver (SCEMS)

## Iterative Algorithm → for a given $P_{in}$

1. Extract "linear" eigenmode (normally)
2. Normalize eigenmode's E-field to  $P_{in}$
3. Calc./apply refr. index perturbation
4. Extract eigenmode of perturbed w/g
5. Repeat (3)-(4) until convergence
6. Calculate mode parameters

### Implementing Step #3

$$\Delta \varepsilon_{r,NL} = \varepsilon_{r,lin} n_2 |\mathbf{E}|^2 / Z_0 \quad (\text{Scalar})$$

$$\Delta \tilde{\varepsilon}_{r,NL} = \frac{3}{4} \sum_{\alpha,\beta}^{x,y,z} \chi_{\mu\alpha\beta\gamma}^{(3)} E_{\alpha}^* E_{\beta} \quad (\text{Tensor})$$

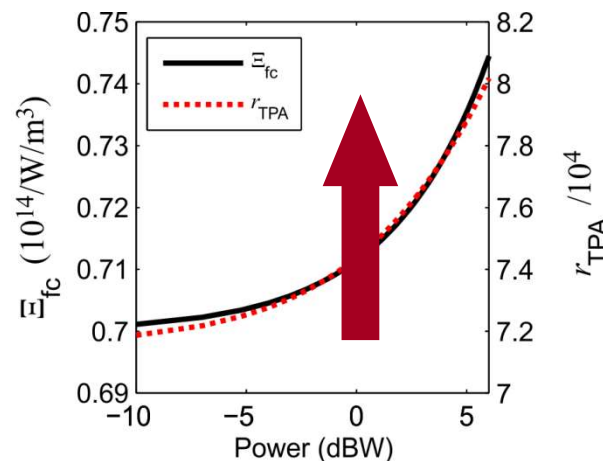
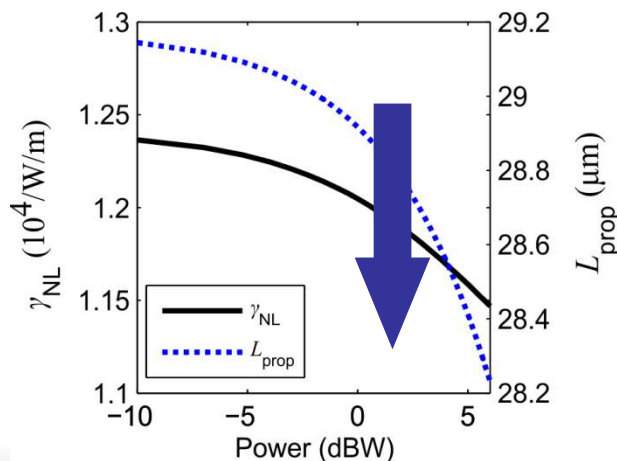
**NLSE:** Mode-parameters now **depend on**  $|A(z,t)|^2$

**SCEMS @ HSP** waveguide  $TM_{00}$  mode:

### Threshold power

$$n_2 P_{in} / A_{eff} > 0.01$$

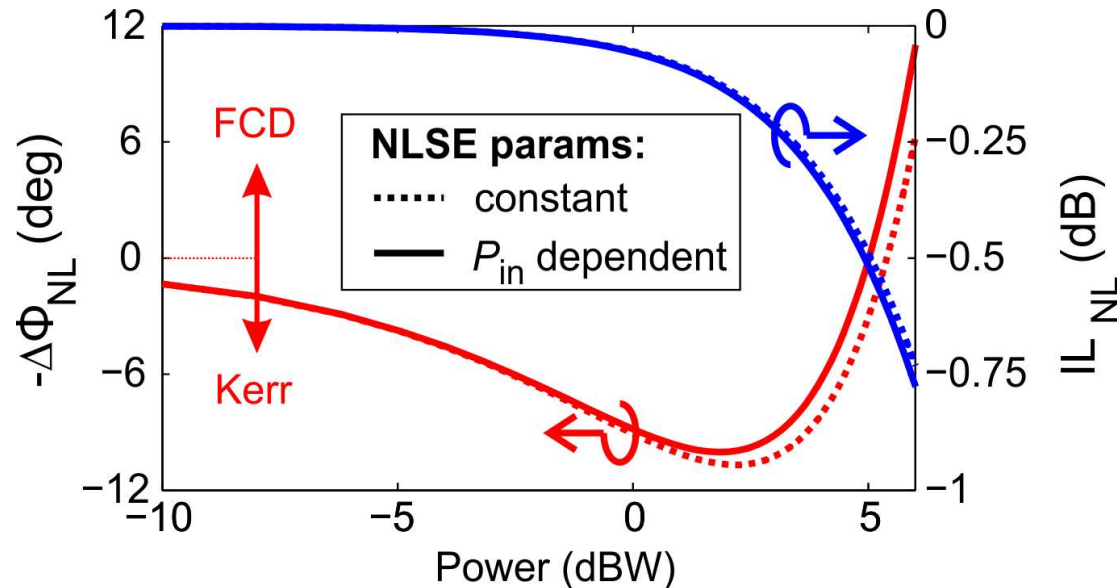
(vs. FCE-threshold)



### FoM drop → $\mathcal{F}$ & $\zeta_{fc}$

Why? Index-contrast at the silicon/dielectric interface decreases. HSP w/g  $TM_{00}$  mode shifts towards the Si-area (from the gap)

## Example CW-NLSE



Optimized HSP waveguide

- ☐ Length equal  $L_{\text{prop}}$
- ☐ Threshold power  $\sim 0\text{dBW}$
- ☒ Reduces Kerr-Phase
- ☒ Extra IL are negligible

### Perturbative effect:

- ☐ Appreciable only at higher-powers
  - ☐ But, “masked” by FCE
- ❖ **Pulsed NLSE:** Similar behavior. Perturbation follows the shape on the pulse → **no spectrum-deformation**

## High-Power Illumination – Prospects & Considerations

### Prospects:

**Materials/Platforms** with higher nonlinearity ( $n_2$ ) and smaller FCE (e.g.  $\tau_{fc}$ )

**Interplay** between linear and nonlinear indices of waveguide materials

### Considerations:

- ❑ **Perturbative NLSE** formulation → Limits breached?
- ❑ **Multimode waveguides** → (more) power-dependent birefringence!
- ❑ **Free-Carrier Effects**
  - Effective Lifetime → Accurate calculation + power-dependence
  - Carrier Density limits → Soref & Bennett model validity range
  - CW case → “worst case”
- ❑ **Technological Concerns**
  - Dielectric-breakdown thresholds
  - Nonlocal effects in metal interfaces (ponderomotive nonlinearity)
  - Thermal generation & exhaustion
  - Material properties at high powers – quintic nonlinear susceptibility?

Nonlinear Plasmonics

# Conclusion & Perspectives



## Concluding remarks & perspectives

**Hybrid silicon-plasmonic waveguides:** an **alternative platform** for photonic components with **nonlinear functionality**.

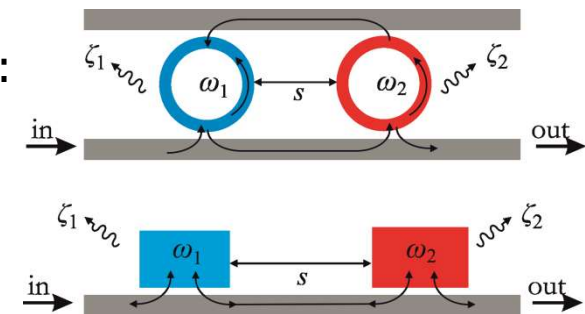
- ❖ **Performance** is catching up with that of Si-based components.
- ✓ Allow for **reduced on-chip interaction lengths**.
- ✓ Extreme **suppression of impairments** due to TPA and FCE.
- ✓ Opens new vistas, e.g. **high-power illumination** in integrated circuits

**Technological steps** that could unlock an **order-of-magnitude boost** in the nonlinear **Figures-of-Merit** (i.e., lower optical power threshold):

- Novel **nonlinear polymers** with  $n_2 \sim 2 \times 10^{-16}$  [m<sup>2</sup>/W], such as PDA/pTS.
- More accurate **control in thin layer deposition**, down to few-nm.

**Future perspectives** for nonlinear hybrid-plasmonics:

- ❖ **Resonant configurations** can further assist the nonlinear response.
- ❖ **Interplay** between all-optical and semiconductor dynamics



*Thank you!*  
*Questions?*

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European Union  
European Social Fund



Co- financed by Greece and the European Union



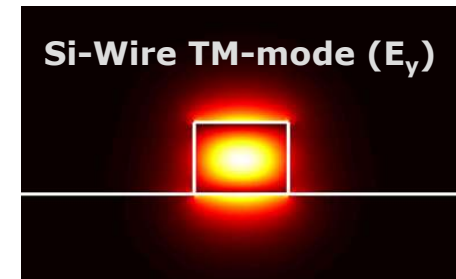
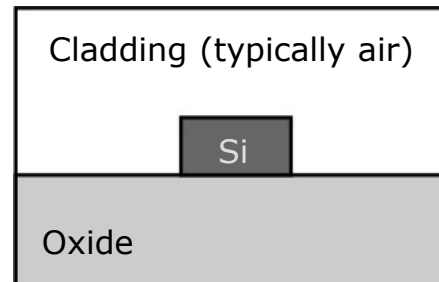
# Backup Material

<http://photonics.ee.auth.gr>

## \* Nonlinear Effects @ Silicon (SOI) Waveguides

### Typical Parameters @ $\lambda=1550\text{nm}$ operation

- $A_{\text{eff}} \sim 0.1\mu\text{m}^2$ 
  - Si-ridge  $\sim 400\times 300\text{nm}^2$
- $\gamma_{\text{NL}} \sim 100\text{m}^{-1}\text{W}^{-1}$ 
  - $n_2 \sim 6\times 10^{-18}\text{m}^2/\text{W}$
  - $r_{\text{TPA}} \sim 0.2$
- $\alpha \sim 1\text{dB/cm}$
- $\tau_{\text{FC,eff}} \sim 1\text{nsec}$  (as low as 10ps)

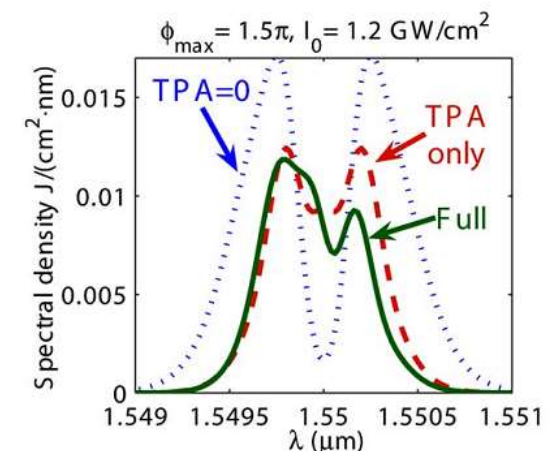


### Typical Performance → for Kerr-induced $\Delta\Phi_{\text{NL}} \sim \pi$

- ❖ **Peak intensity**  $\sim 1\text{GW}/\text{cm}^2 \rightarrow \sim 1\text{W}$  @  $A_{\text{eff}} \sim 0.1\mu\text{m}^2$
- ❖ **Waveguide length**  $\sim 1\text{cm}$
- ✓ **Dispersion** → typically negligible

### Issues of nonlinear SOI w/g:

- × **TPA** → more attenuation limits Kerr-effect
- × **FCA** → even more attenuation...
- × **FCD** → large effect + opposite-sign to Kerr



*Spectral broadening (due to SPM)  
reduction by TPA & FC-Effects  
Yin & Agrawal, Opt. Lett., 2007*

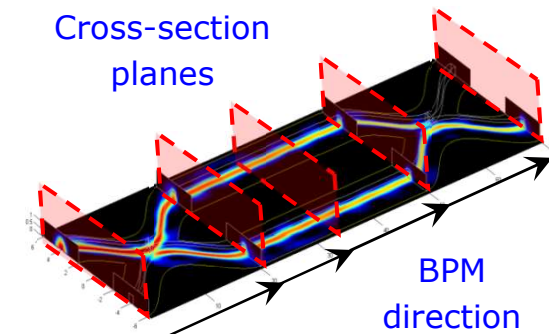
Nonlinear/CW Applications

# Beam Propagation Method

## Validation against the BPM: basic concepts

### A full-3D treatment of the problem

- Field-envelope propagation with a stepping algorithm
- Cross-section discretized with 2D finite elements
- Ideal for longitudinal structures
- Spectral method → CW radiation



### Modeling Propagation under Nonlinear Effects

The waveguide's **refractive-index profile is modified** at each  $\Delta z$ -step by the **Kerr**, **TPA** and **FCE** perturbations that are dependent on  $\mathbf{E}(x,y,z)$

$$\mathbf{D} = \varepsilon_0 \varepsilon_{r,\text{lin}} \mathbf{E} + \mathbf{P}_{3o} + \mathbf{P}_{fc} = \varepsilon_0 \left( n_0^2 + \underbrace{\Delta \tilde{\varepsilon}_{r,3o}}_{\text{Kerr \& TPA}} + \underbrace{\Delta \varepsilon_{r,fc}}_{\text{FCE}} \right) \mathbf{E} = \varepsilon_0 \left( n_0^2 + \Delta \tilde{\varepsilon}_{r,T} \right) \mathbf{E}$$

- Results in an **overall refractive-index modification**
- **Iterative trapezoidal-rule algorithm** for numerical stability

$$\Delta \tilde{\varepsilon}_{r,T}(x, y, z)$$

- Rule-of-thumb for BPM step-size
- Step is adaptively-set as power decreases

$$\Delta z < \frac{\lambda / 40}{\max\{|\sqrt{\Delta \varepsilon_{r,T}}|\}}$$

## Validation against the BPM: implementation of $\chi^{(3)}$ , TPA&FCE

### Implementation of $\chi^{(3)}$ nonlinear susceptibility (Kerr & TPA)

- Index-modification is a **2<sup>nd</sup> rank complex tensor** (e.g. a 3x3 matrix)
  - ✓ Accounts for **hybrid-modes** and **tensor-anisotropy in  $\chi^{(3)}$**  (e.g. silicon)
  - ✓ Requires **fully anisotropic BPM formulation**

$$\Delta\tilde{\epsilon}_{r,30}[\mu,\gamma] = 0.75 \sum_{\alpha} \sum_{\beta} \chi_{\mu\alpha\beta\gamma}^{(3)} E_{\alpha}^* E_{\beta} \quad \chi_{\mu\alpha\beta\gamma}^{(3)}(x,y)$$

- ✓ Reduces to simpler form for **isotropic  $\chi^{(3)}$**

$$\Delta\tilde{\epsilon}_{r,30}[\mu,\gamma] = 0.5 \chi_c E_{\mu}^* E_{\gamma} \delta_{\mu\gamma} + 0.25 \chi_c E_{\mu}^* E_{\gamma} \quad \chi_c(x,y) = \frac{4}{3} \times \frac{n_0^2 n_2}{Z_0} (1 + jr_{\text{TPA}})$$

### Implementation of Free-Carrier Effects (FCD & FCA)

- Index-modification is a **scalar complex** proportional to the **FC-density generated by TPA** (+lifetime).

$$\Delta\epsilon_{r,\text{fc}} = 2n_0 \times \Delta u_{\text{fc}}(x,y)$$

## Validation against the Beam Propagation Method

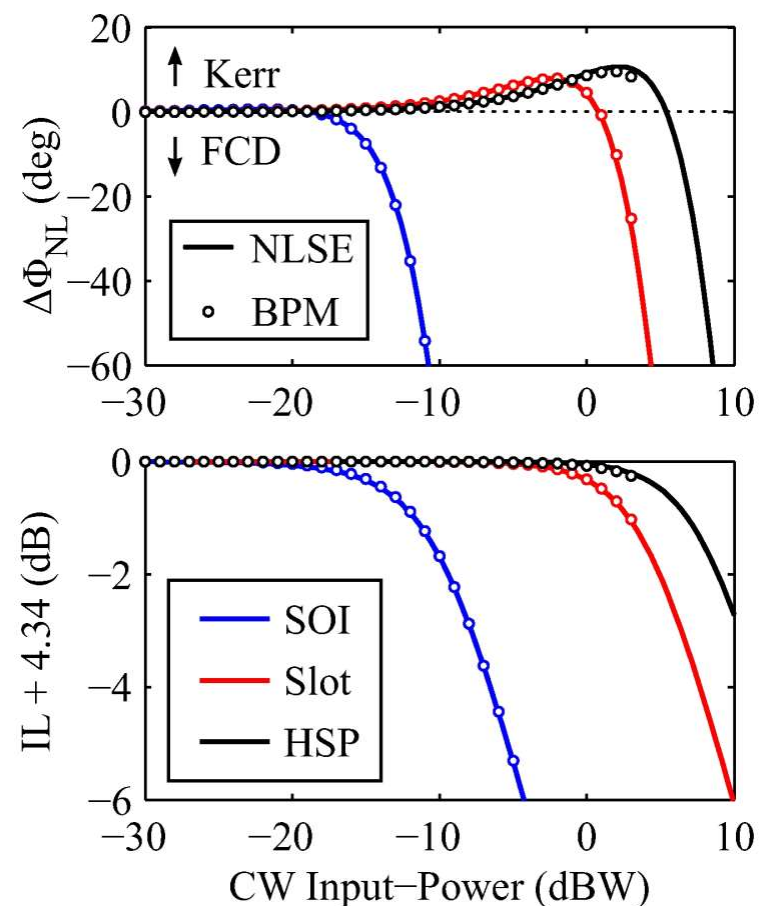
**Comparison:** Overall **phase-shift** ( $\Delta\Phi_{NL}$ ) and **nonlinear insertion losses** ( $IL_{NL}$ ) for the three  $L_{prop}$ -long waveguides.

- Input-profile & reference-index from **mode-solver**.
- Output  $\Delta\Phi_{NL}$  & IL are calculated with **overlap-integrals** on the input-mode.

**BPM vs. NLSE:** Only needs the material properties and waveguide geometry, but is restricted to CW or quasi-CW.

**Attention:** The **local FC-density**  $N(x,y)$  might exceed the validity-limit of the Soref-Bennett model ( $10^{26}/m^3$ ) for powers where the **spatially-averaged FC-density**  $\langle N \rangle_k$  used in the NLSE is much lower.

Excellent agreement!



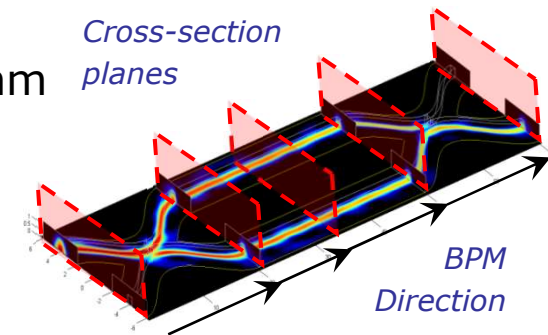


## Design Method #2: Nonlinear Beam Propagation Method

*A more rigorous full-3D treatment of the problem*

### Beam Propagation Method (BPM)

- ❑ Field-envelope propagation with a stepping algorithm
- ❑ Cross-section discretized with 2D finite elements
- ✓ Ideal for longitudinal structures
- ✓ Spectral method → CW radiation



### Modeling Propagation under Nonlinear Effects

- ❑ Cross-section **refractive-index profile** → **Modified** @ each  $\Delta z$ -step
  - ❖ **Kerr**, **TPA** and **FC-effects** with respect to the **electric-field intensity**

$$\mathbf{D} = \epsilon_0 \epsilon_{r,\text{lin}} \mathbf{E} + \mathbf{P}_{\text{NL}} + \mathbf{P}_{\text{FC}} = \epsilon_0 \tilde{\epsilon}_r \mathbf{E} = \epsilon_0 \left( \epsilon_{r,\text{lin}} + \Delta\epsilon_{r,\text{NL}} + \Delta\epsilon_{r,\text{FC}} \right) \mathbf{E}$$

- ❑ **Scalar** or **Tensor** implementation for refractive-index modification
- ❑ **Iterative trapezoidal-rule algorithm** → numerical **stability** @ larger  $\Delta z$

## Design Method #2: BPM-Implementation of NL Effects

❖ **3D-BPM** → Accounts for **Heterogeneity** in (x,y,z) for **E**, **n** &  $\chi^{(3)}$

**Implementation of  $\chi^{(3)}$  nonlinear susceptibility** (Kerr & TPA)

**Scalar** → Index-modification is a **scalar value**

$$\Delta\epsilon_{r,NL} = \epsilon_{r,lin} n_2 |\mathbf{E}|^2 / Z_0$$

✖ Does not account for vector-nature

$r_{TPA}$  included in  $n_2$

**Tensor** → Index-modification is a **2<sup>nd</sup> rank tensor**

✓ Accounts for **hybrid-modes** and **tensor-anisotropy** in  $\chi^{(3)}$  (e.g. silicon)

✓ Requires **fully anisotropic BPM** formulation



$$\Delta\epsilon_{r,NL} \rightarrow \Delta\tilde{\epsilon}_{r,NL}[\mu, \gamma] = \sum_{\alpha} \sum_{\beta} \chi_{\mu\alpha\beta\gamma}^{(3)} E_{\alpha}^* E_{\beta} \quad \mu, \alpha, \beta, \gamma \rightarrow \{x, y, z\}$$

✓ Reduces to simpler form for **isotropic  $\chi^{(3)}$**

$$\Delta\tilde{\epsilon}_{r,NL}[\mu, \gamma] = 0.5\chi_c E_{\mu}^* E_{\gamma} \delta_{\mu\gamma} + 0.25\chi_c E_{\mu}^* E_{\gamma} \quad \chi_c = \frac{4}{3} \times \frac{\epsilon_{r,lin} n_2}{Z_0} (1 + jr_{TPA})$$

**Implementation of Free-Carrier Effects** (FCD & FCA)

Introduced as a **complex scalar index-modification**, proportional to the number of FCs generated by TPA (+lifetime).

$$\Delta\epsilon_{r,FC} = 2n_{lin} (\Delta n_{FC} + j\Delta a_{FC} / 2k_0)$$

Free-Carrier Effects in Silicon Waveguides

# Implementing the FCEs

## Backup: Free-Carrier Effects Modeling

**Electric Displacement:**  $\mathbf{D} = \varepsilon_0 \varepsilon_{r,\text{lin}} \mathbf{E} + \mathbf{P}_{\text{FC}}$

**Electric Polarization** (due to FCs):  $\mathbf{P}_{\text{FC}} = \varepsilon_0 \Delta \varepsilon_{r,\text{FC}} \mathbf{E}$

**Relative Dielectric Constant:**  $\Delta \varepsilon_{r,\text{FC}}(x, y) = 2n_{\text{lin}} (\Delta n_{\text{FC}} + j \Delta a_{\text{FC}} / 2k_0)$

- (only for materials with TPA≠0)

**Real** (phase) & **Imaginary** (loss) parts:

- Soref-model cross-sections

$$\begin{aligned} \Delta n_{\text{FC}} &= +\sigma_a N \\ \Delta a_{\text{FC}} &= -\sigma_n^e N - (\sigma_n^h N)^{0.8} \end{aligned}$$

**Free-Carrier Density:**

- Rate Equation

$$\frac{dN}{dt} = G - \frac{N}{\tau_{\text{FC,eff}}} \xrightarrow{\text{CW}} N = G \times \tau_{\text{FC,eff}}$$

**TPA &  $|E|^2$  dependence:**

- from all modes' contributions

$$G \propto \frac{1}{A_{\text{TPA}} h f} \text{Imag} \left\{ \sum_{m,n}^{1 \dots N} (2 - \delta_{mn}) \gamma_{mn} |A_m A_n|^2 \right\}$$

**FCE parameters ( $G$  &  $N$ )** are inserted:

- (1) NLSE → “effective” value weighted over xy-plane
- (2) 3D-BPM → a function of (x,y) for the “local”  $|E|$  and TPA

A Simpler Approach

# Scalar NLDCS

## Backup: NL-DCS w/ Coupled-waveguide Eqs. + SPM

### A heuristic approach:

Coupled-waveguide formulation + **SPM term**

Left WG	$\frac{\partial A_L}{\partial z} = -\frac{a}{2} A_L + j \frac{\pi}{2L_c} A_R + j \gamma_{NL}  A_L ^2 A_L$
Right WG	$\frac{\partial A_R}{\partial z} = -\frac{a}{2} A_R + j \frac{\pi}{2L_c} A_L + j \gamma_{NL}  A_R ^2 A_R$
	<div style="display: flex; justify-content: space-around; width: 100%;"><div>Losses</div><div>Coupling</div><div>SPM</div></div>

### Parameters are common for the two waveguides

- ❑ **Losses ( $a$ )** → From single-waveguide analysis
- ❑ **Nonlinearity ( $\gamma_{NL}$ )** → From single-waveguide analysis
- ❑ **Beating Length ( $L_c$ )** → From super-mode  $\Delta n_{\text{eff}}$